

Scientific Notation

Scientific notation is commonly used to represent very large or very small numbers in a convenient way.

Scientific notation is written in the following format using a power with base 10.

$m \times 10^b$

m represents the mantissa.
 $1 \leq m < 10$
The mantissa **MUST** be equal to or greater than 1 but less than 10.

b is the exponent on a power with base 10.
Very large numbers will have a positive exponent.
Very small numbers will have a negative exponent.

To write a number in scientific notation:

- Move the decimal to a position immediately to the right of the first nonzero digit. Scan the number from left to right.
- Count the number of place values you had to move the decimal point. This is the value of the exponent.
 - If you moved the decimal point to the **left**, make the **exponent positive**.
 - If you moved the decimal point to the **right**, make the **exponent negative**.
- Drop all trailing or leading zeroes. The remaining number is the mantissa.
- Write the number in scientific notation as a product of the mantissa and power with base 10.

Examples:

1. Write 450,900,000,000 in scientific notation.

4.50,900,000,000



Step 1: Move the decimal to a position immediately to the right of the first nonzero digit. Scan the number from left to right.

4.50,900,000,000

Step 2: Count the number of place values the decimal point was moved. In this case, the decimal point was moved 11 spaces to the left. Therefore, the exponent on base 10 will be positive 11.

4.50,900,000,000

Step 3: Drop the trailing zeros to get the correct mantissa.

4.509×10^{11}

Step 4: Write the number in scientific notation as a product of the mantissa (from step 3) and power of 10 with correct exponent (from step 2).

2. Write 0.0000563 in scientific notation.

$0.00005.63$



Step 1: Move the decimal to a position immediately to the right of the first nonzero digit. Scan the number from left to right.

$0.\underline{00005}.63$

Step 2: Count the number of place values the decimal point was moved. In this case, the decimal point was moved 5 spaces to the right. Therefore, the exponent on base 10 will be *negative 5*.

$~~00000~~5.63$

Step 3: Drop the leading zeros to get the correct mantissa.

5.63×10^{-5}

Step 4: Write the number in scientific notation as a product of the mantissa (from step 3) and power of 10 with correct exponent (from step 2).

Practice Questions:

1. Write 1,705,000 in scientific notation. [Answer: 1.705×10^6]
2. Write 0.0000807 in scientific notation. [Answer: 8.07×10^{-5}]

Addition and Subtraction with Scientific Notation

To ADD or SUBTRACT numbers written in scientific notation:

1. The numbers **MUST have the same exponent** on the powers of 10.
 - To **increase an exponent** in scientific notation, move the decimal point in the mantissa to the **left** the same number of times that you would like to increase the exponent. (For example, to increase the exponent by 2, add 2 to the exponent and move the decimal point in the mantissa to the left two times).
 - To **decrease an exponent** in scientific notation, move the decimal point in the mantissa to the **right** the same number of times that you would like to decrease the exponent. (For example, to decrease the exponent by 3, subtract 3 from the exponent and move the decimal point in the mantissa three times to the right).
2. When all numbers in scientific notation have powers with the same exponent, **add or subtract the mantissa(s). Keep the power the same.**
3. **If necessary, adjust the mantissa and exponent** to put the final answer in proper scientific notation. Remember that the mantissa **MUST** be equal to or greater than 1 but less than 10.

TIP:

Generally, it is best to change all numbers in scientific notation to the power of 10 with the HIGHEST exponent. This eliminates the extra step of putting the final answer back to proper scientific notation.

Example:

Add 5.89×10^4 and 9.5×10^6 .

Step 1: The powers, 10^4 and 10^6 , have different exponents. We need to make these the same.

Method 1: Re-write 9.5×10^6 as mantissa $\times 10^4$

$9.5 \times 10^6 = 950 \times 10^4$ Decrease the exponent by 2; move the decimal point twice to the right in the mantissa.

Step 2: Add the mantissas. Keep the powers the same.

$$\begin{aligned} 5.89 \times 10^4 + 950 \times 10^4 &= (5.89 + 950) \times 10^4 \\ &= 955.89 \times 10^4 \end{aligned}$$

Step 3: Adjust the mantissa and exponent to put the answer in proper scientific notation.

955.89 needs to be adjusted so that the mantissa is greater than or equal to 1 and less than 10.

Move the decimal point in the mantissa two times to the left and add 2 to the exponent.

$$955.89 \times 10^4 = 9.5589 \times 10^{4+2} = 9.5589 \times 10^6$$

←
Final answer is 9.5589×10^6 .

Method 2: Re-write 5.89×10^4 as mantissa $\times 10^6$

$5.89 \times 10^4 = 0.0589 \times 10^6$ Increase the exponent by 2; move the decimal point twice to the left in the mantissa.

Step 2: Add the mantissas. Keep the powers the same.

$$\begin{aligned} 0.0589 \times 10^6 + 9.5 \times 10^6 &= (0.0589 + 9.5) \times 10^6 \\ &= 9.5589 \times 10^6 \end{aligned}$$

Step 3: The mantissa, 9.558, is greater than 1 and less than 10. The answer is in proper scientific notation.

Final answer is 9.5589×10^6 .

Example 2:

Subtract 9.53×10^8 and 1.2×10^7 .

Step 1: The powers, 10^8 and 10^7 , have different exponents. We need to make these the same.

Method 1: Re-write 9.53×10^8 as mantissa $\times 10^7$

$$9.53 \times 10^8 = 95.3 \times 10^7 \quad \text{Decrease the exponent by 1; move the decimal point once to the right in the mantissa.}$$

Step 2: Subtract the mantissas. Keep the powers the same.

$$\begin{aligned} 95.3 \times 10^7 - 1.2 \times 10^7 &= (95.3 - 1.2) \times 10^7 \\ &= 94.1 \times 10^7 \end{aligned}$$

Step 3: Adjust the mantissa and exponent to put the answer in proper scientific notation.

94.1 needs to be adjusted so that the mantissa is greater than or equal to 1 and less than 10.

Move the decimal point in the mantissa once to the left and add 1 to the exponent.

$$94.1 \times 10^7 = 9.41 \times 10^{7+1} = 9.41 \times 10^8$$

←
Final answer is 9.41×10^8 .

Method 2: Re-write 1.2×10^7 as mantissa $\times 10^8$

$$1.2 \times 10^7 = 0.12 \times 10^8 \quad \text{Increase the exponent by 1; move the decimal point once to the left in the mantissa.}$$

Step 2: Subtract the mantissas. Keep the powers the same.

$$\begin{aligned} 9.53 \times 10^8 - 0.12 \times 10^8 &= (9.53 - 0.12) \times 10^8 \\ &= 9.41 \times 10^8 \end{aligned}$$

Step 3: The mantissa, 9.41, is greater than 1 and less than 10. The answer is in proper scientific notation.

Final answer is 9.41×10^8 .

Practice Questions:

3. Add. $1.567 \times 10^3 + 3.2 \times 10^4$ [Answer: 3.3567×10^4]
4. Subtract. $7.85 \times 10^5 - 4.5 \times 10^3$ [Answer: 7.805×10^5]

Multiplication and Division with Scientific Notation

To **MULTIPLY** numbers written in scientific notation:

1. **Multiply the mantissas.**
2. **Add the exponents** on powers of 10.
3. **If necessary, adjust the mantissa and exponent** to put the final answer in proper scientific notation.

Note: The numbers do NOT need to have the same exponent on the powers of 10.

Example:

Multiply 3.67×10^8 and 1.3×10^3 .

$$(3.67 \times 10^8)(1.3 \times 10^3) = (3.67 \times 1.3) \times 10^{8+3}$$

Step 1: Multiply the mantissas.

Step 2: Add the exponents.

$$= 4.771 \times 10^{11}$$

Answer is in proper scientific notation.

To **DIVIDE** numbers written in scientific notation:

1. **Divide the mantissas.**
2. **Subtract the exponents** on powers of 10.
3. **If necessary, adjust the mantissa and exponent** to put the final answer in proper scientific notation.

Note: The numbers do NOT need to have the same exponent on the powers of 10.

Example:

Divide 9.5×10^8 and 5×10^4 .

$$(9.5 \times 10^8) \div (5 \times 10^4) = (9.5 \div 5) \times 10^{8-4}$$

Step 1: Divide the mantissas.

Step 2: Subtract the exponents.

$$= 1.9 \times 10^4$$

Answer is in proper scientific notation.

Practice Questions:

5. Multiply 1.87×10^3 by 5.193×10^4 . [Answer: 9.71091×10^7]
6. Divide 7.707×10^5 by 2.1×10^2 . [Answer: 3.67×10^3]